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PROPAGATION OF A PLANAR SHOCK THERMAL WAVE IN A NONLINEAR MEDIUM

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Conditions are analyzed for a strong discontinuity of a thermal field in a nonlinear medium, possessing thermal relaxation. The solution of the heat-transfer equations is obtained in the region between the mobile boundaries and the front of the strong discontinuity.

1. Analysis of Strong Discontinuity Conditions of a Thermal Field. The heat-transfer equations in a medium with a relaxing thermal flow [1, 2] are written in the following form for one-dimensional processes with planar symmetry

$$\rho h_t + q_x = 0, \quad \rho,\ \gamma - \text{const}, \tag{1}$$

$$L_x + \gamma q_t + q = 0, \quad h = \int_0^T c_p(T)\,dT, \quad L = \int_0^T \lambda(T)\,dT. \tag{2}$$

Shock thermal waves can be generated in nonlinear media possessing thermal relaxation [2, 3]. In particular, an important object of application of the heat-transfer model (1), (2) are thermal perturbations in liquid helium [4, 5]. It is well known that second sound shock waves can occur in liquid helium at temperature 1.2K < T < 2.0K; the physical analysis of this effect and a bibliography are given in [5, 6]. In the presence of relaxation properties of the medium surfaces of strong discontinuity are also formed in other physicomechanical processes, for example, in liquid filtration [7]. This question is discussed in [2].

To obtain conditions of dynamic compatibility at the strong discontinuity line of the thermal field the energy conservation law must be selected in integral form, and then the method of [8, 9] must be applied:

$$N\{\rho h\} = \{q\}, \quad N = dx_j/dt. \tag{3}$$

Here the brackets denote the jump of the corresponding functions during transition through the strong continuity line $x = x_j(t)$.

Also possible are statements of the heat-transfer problem, in which the single condition (3) is insufficient to guarantee uniqueness of the solution, and, according to [9], an additional relation is required at the discontinuity.

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Consider, for example, a thermal process with $x \in [x_b, x_j]$. If only one boundary condition is known at the boundary $x = x_b(t)$, for unique determination of $x = x_j(t)$ one requires an additional condition at the discontinuity. To obtain this condition consider Eq. (2), following from the integral equation

$$\int_{x_1}^{x_2} (\gamma q |_{t_1}^{t_2})\, dx + \int_{t_1}^{t_2} (L)_{x_1}^{x_2}\, dt = -\int_{t_1}^{t_2}\int_{x_1}^{x_2} q dx dt, \tag{4}$$

where x_1, x_2 fix the specified volume, and t_1, t_2 are arbitrary moments of time. Applying the method of [8, 9], we obtain from (4) the additional condition at the strong discontinuity of the thermal field

$$N\{\gamma q\} = \{L\}. \tag{5}$$

It follows from relations (3), (5) that $N \neq 0$, therefore we consider here a strong discontinuity, whose displacement velocity is discontinuous, retaining its unchanging sign. We further assume that $\{\rho\} = 0$, $\{\gamma\} = 0$.

Conditions (3), (5) can be represented in form of the relations

$$\begin{gathered} q_j^* - \rho N h_j^* = q_j - \rho N h_j = Q(x_j, t) \equiv Q_j,\ L_j = L(T_j),\quad h_j = h(T_j), \\ \gamma N q_j^* - L_j^* = \gamma N q_j - L_j = \Lambda(x_j, t) \equiv \Lambda_j,\quad T_j = T(x_j, t), \end{gathered} \tag{6}$$

characterizing the discontinuity at the line $x = x_j(t)$ of the quantities $Q = q - \rho NL$, $\Lambda = \gamma Nq - L$, having, respectively, the dimensions of specific thermal flow and thermal flow gradient (power gradient). We note that $h(T)$, $L(T)$ are discontinuous functions of temperature, while the brackets $\{h\}$, $\{L\}$ are due to the discontinuity $\{T\} \neq 0$.

It is easily seen that if λ, c_p are continuous, it then follows from Eqs. (3) and (5) that $N^2 = w^2 \equiv \lambda/\gamma\rho c_p$, i.e., the strong discontinuity degenerates into a continuous thermal wave.

The following conclusions are drawn from Eq. (6): 1. If Q_j = const, Λ_j = const, the strong discontinuity of the thermal field, displaced toward a constant velocity, degenerates into a weak discontinuity, a continuous thermal wave. 2. If there exists a strong discontinuity, it moves according to the homogeneous thermal field $T^* \equiv$ const, $q^* \equiv 0$ with a variable velocity. 3. A strong discontinuity can be displaced with a constant velocity $N \neq w \equiv$ const only by the nonuniform thermal field $T = T^*(x, t)$, $q = q^*(x, t)$.

Conditions (3), (5) are written in the form

$$q_j - q_j^* = \rho N(h_j - h_j^*), \tag{7}$$

$$L_j - L_j^* = \gamma N(q_j - q_j^*). \tag{8}$$

Substituting (7) into (8), we obtain

$$h_j = h_j^* + (M_j - M_j^*)/\rho\gamma N^2,\quad N \neq 0,\quad L(T) = M[h(T)]. \tag{9}$$

It is assumed that the thermal field is given ahead of the front of the strong discontinuity, and that the velocity $N = N(t)$ of the displaced discontinuity is known. Relations (7), (9) are then algebraic equations for q_j, h_j, and time appears in them as a parameter. The functions $q_j^*(t)$, $T^*_j(t)$, $N(t)$ are assumed to be continuous for $t \in [0, t_k]$, $0 < t_k < \infty$; considering the thermal process at an infinite intermediate time, we assume that these functions are both continuous and bounded. We further use the notation $t \in [0, t_k]$, keeping in mind both these cases. Thus, we put

$$\begin{gathered} 0 < N_1 \leqslant N(t) \leqslant N_2 < \infty,\quad t \in [0, t_k], \\ 0 \leqslant T_1^* \leqslant T_j^*(t) \leqslant T_2^* < \infty,\quad -\infty < q_1^* \leqslant q_j^*(t) \leqslant q_2^* < \infty. \end{gathered} \tag{10}$$

Equation (9) is convenient for calculating h_j by the method of successive approximations. Sufficient convergence conditions of this iteration procedure to a unique solution in the interval $[T', T'']$ consist of the following [10]: 1. The right-hand side of (9) satisfies the inequalities

$$\begin{gathered} h' = h(T') \leqslant h_j^* + [M(h_j) - M(h_j^*)]/\rho\gamma N^2 \leqslant h(T'') = h'', \\ [T', T''] \subseteq [T_1, T_2],\ T_j(t) \in [T_1, T_2],\ h_j(t) \in [h_1, h_2], \end{gathered} \tag{11}$$

where T' and T" depend parametrically on t. 2. The following inequality is satisfied at each moment of time

$$w^2(T_j) < N^2(t), \ T' < T_j < T''. \tag{12}$$

We carry out some estimates allowing to infer to what extent conditions (11), (12) are satisfied. We take into account that h(T), L(T), and M(h) are monotonically increasing functions of their arguments, and we denote by $T = \tau(h)$ the inverse function to $h = h(T)$. For the continuous function $w^2(T)$ we specify temperature intervals in which it increases or decreases monotonically, i.e., there exists an inverse function $T = \theta(w^2) < \infty$. Taking into account Eq. (10), instead of (11), (12) we choose the cruder conditions

$$h_1 \leqslant h_j^* + (M_j - M_j^*)/\rho\gamma N^2 \leqslant h_2, \ w^2(T_j) < N_1^2. \tag{13}$$

Let $w^2(T)$ be a monotonically increasing function. From (13) we then obtain at any moment of time, and as upper temperature boundary behind a strong discontinuity one can take

$$T_2 = \theta(N_1^2) - \varepsilon_2 > 0, \ \varepsilon_2 > 0. \tag{14}$$

This number ε_2 can always be found, since $\theta(w^2)$ is continuous. Further approximations are obtained by means of (10), (14), and some uncomplicated transformations of inequality (13). If $T_2 > T_1 > 0$, and the following inequality is valid

$$h_2 > h_1 + (M_2/\rho\gamma N_1^2), \tag{15}$$

then for h_2^* the following estimate holds

$$0 < h_2^* < h_2 - (M_2/\rho\gamma N_1^2). \tag{16}$$

It is easily seen that in the presence of (15), (16), by a suitable choice of $N_2 > N_1$ one satisfies the inequality

$$h_2 - (M_2/\rho\gamma N_1^2) > h_1 + (M_2^*/\rho\gamma N_2^2), \ M_2^* = M(h_2^*), \tag{17}$$

and the following estimate is then obtained for h_1^*:

$$h_1 + (M_2^*/\rho\gamma N_2^2) \leqslant h_1^* < h_2^*. \tag{18}$$

It follows from Eqs. (16)-(18) that, in particular, the boundaries of the temperature intervals must be located as follows: $T_1 < T_1^* < T_2^* < T_2$.

Thus, in the case of a monotonically increasing function $w^2(T)$ Eq. (9) has at any moment of time a unique root in the interval $[h_1, h_2]$ if the inequalities (10), (14)-(18) are satisfied.

Let $w^2(T)$ be a monotonically decreasing function. At any moment of time we then obtain from (13) the inequality $T_j = \theta(w^2{}_j) > \theta(N_1^2)$, according to which one can take

$$T_1 = \theta(N_1^2) + \varepsilon_1 > 0, \ \varepsilon_1 > 0. \tag{19}$$

In this case there exists an ε_1, such that $w^2(T_1) > w^2(T_2) > 0$. If Eq. (15) is satisfied for $T_2 > T_1 > 0$ values, then similarly to the preceding case, by using (19) we can verify the validity of estimates (16)-(18), guaranteeing for Eq. (9) the existence of a unique root in the interval $[h_1, h_2]$ for any $t \in [0, t_k]$.

We provide an estimate of the interval [T', T"] in (11), (12) at some fixed moment of time. If $w^2(T)$ is a monotonically increasing function, we select $h' = h_j^* - (M_j^*/\rho\gamma N^2)$, $h'' = h(\theta_j)$, $\theta_j = \theta(N^2)$, and require that $N(t)$, $T_j^*(t)$ satisfy the inequalities $h' \geq 0$, $h'' > h_j^* + [L(\theta_j) - L_j^*]/\rho\gamma N^2$. Equation (9) then has a single root $h_y(t)$ in the interval [h', h"].

If $w^2(T)$ is a monotonically decreasing function, we select $h' = h(\theta_j)$, $T' = \theta(N^2)$, and require that the inequality $0 < h(\theta_j) < h_j^* + [L(\theta_j) - L_j^*]/\rho\gamma N^2$ be satisfied. There then exist a value $T'' = \tau(h'') > T'$, $h'' = h_j^* + [L(\theta_j)]/\rho\gamma N^2$, for which conditions (11), (12) are valid, while Eq. (9) has a single root in the interval [h', h"].

Differentiating Eq. (9) with respect to time, we obtain the expression

$$c_p(T_j)\nu_j\dot{T}_j = c_p(T_j^*)\nu_j^*\dot{T}_j^* - 2\frac{\dot{N}}{N}(h_j - h_j^*), \quad \nu(t, T) = 1 - \frac{w^2(T)}{N^2(t)}, \tag{20}$$

making it possible to draw several qualitative conclusions. According to (12) we further assume that $\nu_j > 0$, and for definiteness we put $N > 0$. If the strong discontinuity propagates with a constant velocity with a nonuniform stationary background $T = T^*(x)$, we conclude from Eq. (20) that for $dT^*/dx > 0$ we have $\mathrm{sgn}\dot{T}_j = \mathrm{sgn}\nu_j^*$, while for $dT^*/dx < 0$ we have $\mathrm{sgn}\dot{T}_j = -\mathrm{sgn}\nu_j^*$. This implies that in this case the temperature increase or decrease behind the front of the strong discontinuity depends on both the direction of the temperature gradient ahead of the front and on the relation between the velocity of the strong discontinuity and the propagation velocity of the continuous thermal wave at $T = T^*$.

If the strong discontinuity propagates with a varying velocity in the "cold" background $T^* = 0$, $q^* = 0$, we then conclude from Eq. (20) that $\mathrm{sgn}\dot{T}_j = -\mathrm{sgn}N$. Consequently, in this case the medium temperature increases behind the front moving with a negative acceleration; the medium temperature decreases behind the front if it moves with a positive acceleration.

2. Transformation of the Heat-Transfer Equations. Based on Eq. (1) we introduce into our treatment the function $\eta = \eta(x, t)$, whose total differential is $d\eta = \rho h dx - q dt$. Taking into account Eq. (3), we conclude that along the strong discontinuity line $d\eta + \psi(t)dt = 0$, where

$$\psi(t) = q_j^* - [h_j^*(q_j - q_j^*)/(h_j - h_j^*)]. \tag{21}$$

We transform the heat-transfer equations (1)-(2) to the new independent variables ξ, t according to the equations

$$d\xi = d\eta + \psi dt,\ \partial/\partial x = \rho h \partial/\partial\xi,\ \partial/\partial t = \partial/\partial t' + (\psi - q)\,\partial/\partial\xi,\ \rho h \neq 0. \tag{22}$$

As a result we obtain the system of equations

$$h_t + (\psi - q)\,h_\xi + hq_\xi = 0,\ \ \gamma q_t + q + \rho h L_\xi + \gamma(\psi - q)\,q_\xi = 0,\ \ L(T) = M(h). \tag{23}$$

The variables ξ, t are suitable for studying nonstationary thermal fields containing shock thermal waves, since the family of straight lines ξ = const in the x, t plane is a family of lines of strong discontinuity [2].

3. A Nonstationary Thermal Field between a Mobile Boundary and a Strong Discontinuity. Let a strong discontinuity propagate with constant velocity in the nonuniform stationary field $T = T^*(x)$, $q^* \equiv$ const. We construct a solution of Eq. (23) for the values $x \in [x_b, x_j]$, acquiring the value $\xi = 0$ behind the line of strong discontinuity, $\xi \in [0, \xi_b]$.

The temperature and the specific thermal flow behind the front of the strong discontinuity are represented in the form of the series

$$T = T_0 + T_\delta(\xi)\,s^\delta,\ \ q = q_\delta(\xi)\,s^\delta,\ \ \delta \geqslant 1,\ \ s = \alpha\exp(-kt),\ \ \alpha > 0,\ \ k > 0,$$
$$T_0 \equiv \mathrm{const}. \tag{24}$$

Here the repeated subscript δ implies summation. Concerning the thermophysical properties of the medium, we assume that $\lambda = \dot{\ell}(T)/\ell(T)$, $c_p(T)$, $\ell(T)$ are analytic functions of temperature. In constructing expansions of these functions in powers of s we apply the equation [11]:

$$F(T) = F_0 + F_\delta(\xi)\,s^\delta,\ \ F_0 = F(T_0),$$
$$F_n(\xi) = \sum_{r=1}^{n} \frac{F^{(r)}(T_0)}{r!} \sum_{m_1+\ldots+m_r=n,\, m_s\geqslant 1} T_{m_1}\ldots T_{m_r},\ \ n \geqslant 1. \tag{25}$$

An equation of similar type is also used in carrying out other operations with series.

The heat-exchange condition at the left boundary: $x = x_b$, $q = q_b$. The equation of motion of the left boundary and the thermal regime on it are related by the equation

$$d\xi_b/ds = \rho h_b d\tilde{x}_b/ds - (\psi - q_b)/ks,\ \ \xi_b(\alpha) = 0;\ \ \tilde{x}_b(s) = x_b(t),\ \ x_b \leqslant x_j, \tag{26}$$

following from Eq. (22).

Ahead of the discontinuity front we have $L(T^*) = q^*(x + \ln k^*)$, $k^* > 0$, $0 < T^*(x) < T^*_\infty$, $T^*_\infty = \lim_{x\to\infty} T^*(x) < \infty$, x [1, ∞). Accordingly $T_j^* = T^*[\tilde{x}_j(s)]$, and we find, by inverting the dependence,

$$l(T_j^*)/l(0) = [k^*(\alpha/s)^{\frac{N}{k}}]^{q^*},\ \ \tilde{x}_j(s) = \frac{N}{k}\ln\frac{\alpha}{s},\ \ 0 < T_j^* < \infty,\ \ l(0) \neq 0.$$

For example:

$$\lambda(T)=\frac{g_0}{(g_1+g_2T)(g_3+g_4T)},\quad T_j^*=\frac{g_1g_3(1-A)}{g_2g_3A-g_1g_4},$$

$$A=\left[k^*\left(\frac{\alpha}{s}\right)^{\frac{N}{k}}\right]^{q_*g_5},$$

$$q^*g_5<0,\ g_3g_4<0,\ g_5=g_1g_4-g_2g_3,\ T_j^*|_{s=0}=T_\infty^*.$$

The case in which the region of variation of x ahead of the front is a finite interval is treated similarly.

Thus, the function ψ in Eq. (21) can be represented by the series

$$\psi=\psi_0+\psi_\delta s^\delta,\quad \delta\geqslant 1,\ \psi_0=q^*[1+h_\infty^*/(h_0-h_\infty^*)].$$

We require that the series (24) formally satisfy the system (23). This leads to the following recurrence linear equations for the series coefficients

$$\frac{dT_n}{d\xi}=u_nT_n+v_nq_n+\dot{\alpha}_{n-1}(\xi),\quad \frac{dq_n}{d\xi}=r_nT_n+w_nq_n+\beta_{n-1}(\xi),\ n\geqslant 1,$$

$$u_n\Delta=\gamma\psi_0c_0kn,\ v_n\Delta=h_0(1-\gamma kn),\ r_n\Delta=-\rho\lambda_0h_0c_0kn,$$

$$w_n\Delta=c_0\psi_0(\gamma kn-1),$$

$$\alpha_{n-1}\Delta=\gamma\psi_0A_{n-1}-h_0B_{n-1},\ \beta_{n-1}\Delta=c_0\psi_0B_{n-1}-\rho\lambda_0h_0A_{n-1},\ A_0=B_0=0,$$

$$A_{n-1}=\sum_{\zeta=1}^{n-1}[kc_\zeta(n-\zeta)T_{n-\zeta}-\varphi_\zeta\dot{T}_{n-\zeta}-h_\zeta\dot{q}_{n-\zeta}],\quad \varphi_\zeta=\sum_{l=0}^{\zeta}c_l(\psi_{\zeta-l}-q_{\zeta-l}),\tag{27}$$

$$B_{n-1}=-\sum_{\zeta=1}^{n-1}[\rho\mu_\zeta\dot{T}_{n-\zeta}+\gamma(\psi_\zeta-q_\zeta)\dot{q}_{n-\zeta}],$$

$$\mu_\zeta=\sum_{l=0}^{\zeta}\lambda_lh_{\zeta-l},\ \Delta=\gamma c_0\psi_0^2-\rho\lambda_0h_0^2\neq 0.$$

Here α_{n-1}, β_{n-1} consist of expansion coefficients with subscripts not exceeding $n-1$. A nonlinearity of thermophysical medium properties occurs in this class of solution, starting with the second approximation. The characteristic equation corresponding to the system (27) has these roots:

$$2m_n^{(i)}\Delta=c_0\psi_0(2\gamma kn-1)\pm\sqrt{D_n},\ D_n=c_0^2\psi_0^2+4\rho\lambda_0c_0h_0^2(\gamma kn-1)kn,\quad i=1,\ 2.\tag{28}$$

The basic case here is that in which $\gamma k>1$, i.e., $D_n>0$, $n\geq 1$, and there are two real roots. Clearly, the variant $D_n\leq 0$ occurs only for finite $n\leq n_0<\infty$ values, since with increasing n we necessarily obtain $D_n>0$. For example, if $\gamma kn_0-1<0$, $n_0\geq 1$, and the c_0, ψ_0, λ_0, h_0, ρ values are such that the case $D_n<0$, $n\leq n_1<\infty$ is possible, then there exists a finite number of complex conjugate roots. This fact does not entail substantial changes in the algorithm of solution construction: the finite number of series terms (24) is then constructed by equations corresponding to complex roots, while the structure of the following expansion terms is the same as for $\gamma k>1$. Similar considerations also apply to the case in which $\gamma kn_2=1$ and $n_2\geq 1$ is an integer. The further calculations are carried out for $\gamma k>1$.

Applying the method of variable constants to the linear inhomogeneous equations (27), we find the general solution in the form

$$T_n=p_n^{(1)}(J_n^{(1)}+\tau_n)\exp(m_n^{(1)}\xi)+p_n^{(2)}(J_n^{(2)}+\varkappa_n)\exp(m_n^{(2)}\xi),\ \gamma k>1,\tag{29}$$

$$q_n = (J_n^{(1)} + \tau_n)\exp(m_n^{(1)}\xi) + (J_n^{(2)} + \varkappa_n)\exp(m_n^{(2)}\xi),\ p_n^{(i)}(m_n^{(i)} - u_n) = v_n,$$

$$J_n^{(1)} = \int_0^\xi \frac{(p_n^{(2)}\beta_{n-1} - \alpha_{n-1})}{p_n^{(2)} - p_n^{(1)}} \exp(-m_n^{(1)}\xi)\,d\xi,$$

$$J_n^{(2)} = \int_0^\xi \frac{\alpha_{n-1} - p_n^{(1)}\beta_{n-1}}{p_n^{(2)} - p_n^{(1)}} \exp(-m_n^{(2)}\xi)\,d\xi.$$

Here τ_n, κ_n, $n \geq 1$, are arbitrary constants, therefore it can be assumed that the expansions (24), (29) formally contain two arbitrary functions of the same argument s.

We indicate the region of variation of $\xi_b(s)$ as a function of the signs of roots of the characteristic equation. If $\Delta > 0$, then for $\psi_0 < 0$, according to (28), we have $m_n^{(i)} < 0$ and for sufficiently large n we have $dm_n^{(i)}/dn < 0$, $i = 1, 2$; implying $\xi_b \in [0, \infty)$. If $\Delta > 0$, $\psi_0 > 0$, we must have $\xi_b \in (-\infty, 0]$. In what follows the function $\xi_b(s)$ is arbitrary. In both cases, for sufficiently large n we have $\exp m_{n+1}^{(i)}\xi / \exp m_n^{(i)}\xi < 1$, $n \geq n_3 \geq 1$, $i = 1, 2$.

For $\Delta < 0$ the roots of different sign $m_n^{(i)} < 0$, $m_n^{(2)} > 0$, therefore there exists no such finite or infinite region of ξ values, such that $\exp m_n^{(i)}\xi$, $i = 1, 2$, are finite for any n 1. In what follows we choose $\Delta > 0$.

Writing down the solution (24), (29) for $\xi = 0$, as well as using an equation of type (25), we represent the two functions appearing in conditions (7), (8) in the form of power series in s:

$$\begin{gathered} q_j = (\tau_\delta + \varkappa_\delta)s^\delta,\ T_j = (p_\delta^{(1)}\tau_\delta + p_\delta^{(2)}\varkappa_\delta)s^\delta,\ q_j^* \equiv \text{const}, \\ L_j = L(T_0) + [\lambda(T_0)T_{\delta j} + b_{\delta-1}]s^\delta,\ h_j = h(T_0) + [c_p(T_0)T_{\delta j} + a_{\delta-1}]s^\delta \\ L_j^* = L(T_\infty^*) + L_\delta^* s^\delta,\ h_j^* = h(T_\infty^*) + h_\delta^* s^\delta. \end{gathered} \tag{30}$$

Here the quantities a_{n-1}, b_{n-1}, $n \geq 1$, consists of expansioncoefficient with subscripts not exceeding $n - 1$.

After substituting (30) into (7), (8), we find in the zeroth approximation:

$$q^* = \rho N(h_\infty^* - h_0),\quad \gamma N q^* = L_\infty^* - L_0. \tag{31}$$

If N is assumed known, an equation of type (9) follows from (31). Also assuming that q^* is known, we have

$$h_0 = h_\infty^* + [\gamma q^{*2}/\rho(M_0 - M_\infty^*)]. \tag{32}$$

The sufficient existence conditions of a unique root h_0 of Eq. (32) are similar to the corresponding conditions for equations of type (9), in which case the requirement $\Delta > 0$ is satisfied. It is further assumed that the quantity q^* is given, while T_0 and N are found from (31), (32).

In the n-th approximation we obtain a system of linear algebraic equations

$$\begin{gathered} \tau_n[1 - \rho N c_p(T_0)p_n^{(1)}] + \varkappa_n[1 - \rho N c_p(T_0)p_n^{(2)}] = \rho N(a_{n-1} - h_n^*), \\ \tau_n[\gamma N - \lambda(T_0)p_n^{(1)}] + \varkappa_n[\gamma N - \lambda(T_0)p_n^{(2)}] = b_{n-1} - L_n^*,\ n \geqslant 1. \end{gathered} \tag{33}$$

Hence one easily finds τ_n, κ_n by the Kramer equations [10]. The determinant of the system (33) is nonvanishing since $m_n^{(1)} \neq m_n^{(2)}$.

Rearranging Eq. (26) and the solution $T(\xi, s)$, $q(\xi, s)$ behind the front of the strong discontinuity, one can determine, arbitrarily within one function $\xi_b(s)$, the coordinate $\tilde{x}_b(s)$ of the left boundary and of the thermal regime on them.

From the convergence behavior of the series (24), (29) we note the following. The series coefficients of the form $[J_\delta^{(i)} \exp(m_n^{(i)}\xi)]\, s^\delta$, $i = 1, 2$, are analytic functions of the argument ξ, vanishing for $\xi = 0$, while for $\tilde{\xi} \in [0, 1)$ the convergence is maximal for $s \in (0, \alpha]$ of the series [12]. This implies that the series converge uniformly for $\xi \in [0, 1)$, $s \in (0, \alpha]$.

Taking into account the comments made above on the behavior of the roots $m_n^{(i)}$, $i = 1, 2$, and applying the method of majorant functions [12], we conclude that series of the type $[\tau_\delta \exp(m_n^{(i)}\xi)]\, s^\delta$, $[\kappa_\delta \exp(m_n^{(i)}\xi)]\, s^\delta$ converge for $\xi \in [0, \varepsilon)$, $s \in (0, \alpha]$, $0 < \varepsilon < 1$, $0 < \alpha < 1$, if $T^*(x)$ is an analytic function of the argument x.

We sum up. At the initial moment of time $t = 0$ and for $x = 0$ we have a strong discontinuity $\{T\} = T(0, \alpha) - T^*(0)$, $\{q\} = q(0, \alpha) - q^*$, coinciding with the mobile boundary $x_b(0) = x_b(\alpha) = 0$. The discontinuity front, on which conditions (7), (8) are satisfied, is displaced with a constant velocity $\tilde{N}$ in the nonuniform stationary field $T = T^*(x)$, $q^* \equiv \text{const}$, $x \geq 0$. The functions $\tilde{x}_b(s)$, $q_b(s)$, corresponding to the mobile boundary region, are determined within one arbitrary function $\xi_b(s)$. The series (24), appearing in Eqs. (29), (32), (33) and representing the temperature and specific thermal flow for $x \in [x_b, x_j]$, $t \geq 0$, converge for $\xi \in [0, \varepsilon)$, $s \in (0, \alpha]$, $0 < \varepsilon < 1$, $0 < \alpha < 1$.

NOTATION

x, a Cartesian coordinate; t, time; ξ, a new independent variable; T, temperature; q, specific thermal flux; λ, thermal conductivity coefficient of the medium; c_p, specific heat capacity; γ, relaxation time of the thermal flux; h, enthalpy; ρ, density; $\tilde{N}$, displacement velocity of strong discontinuity; w, propagation velocity of small thermal perturbations; L(T) and M(h), auxiliary functions; $m_n^{(i)}$, roots of the characteristic equation; and τ_n, κ_n, arbitrary constants. The subscripts are the following: b, value at the mobile boundary of the region $x = x_b(t)$; j, value on the strong discontinuity line $x = x_j(t)$; *, thermal field parameters ahead of the strong discontinuity; ∞, function value at $x \to \infty$, independent variables as subscripts denote partial differentiation; a dot over the function sign denotes ordinary differentiation; and δ, a summation subscript.

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